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COMMON FIXED POINT THEOREM OF FOUR MAPPING

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ABSTRACT

Srivastava [12],[9], Dubey & Dubey [3], Bhola & Sharma[1] Pandey &Dubey [6]; Considered three self mappings and obtained a unique common fixed point. Here we generalized the contraction used by above authors for five maps and obtained a unique fixed point.

KEYWORDS: Fixed Point Theorems, Mappings, Contraction Mappings, Continuous Mappings, Complete Metric Space

INTRODUCTION

Srivastava [9] used the inequality as follows

$$d(E_x, f_y) \le C_1\left[\frac{d(T_x,E_x)d(T_y,F_y)}{d(T_x,T_y)}\right] + C_2\left[\frac{d(T_x,F_y)d(T_y,F_y)}{d(T_x,F_y)}\right] \quad [1.1]$$

$$+ C_3\left[d(T_x,F_y) + d(T_y,F_y)\right] + C_4\left[d(T_x,F_y) + d(T_y,E_x)\right]$$

$$+ C_5\left[d(T_x,T_y) + d(E_x,F_y)\right] + C_6 d(T_x,T_y);\ \forall\, x, y \in X$$

$$T_x \ne T_y$$

Then, E, F, T have a unique common fixed point if

$$C_i \ge 0;\ \ 0 \le C_1 + 2(C_2 + C_3 + C_4 + C_5 + C_6) \le 1,$$

$$0 \le 2(C_4 + C_5 + C_6) \le 1;$$

Dubey &Dubey [3], have proved a fixed point theorem for three maps S,T &I of a complete metric space (X,d) satisfying:

$$d(S_x,T_y) \le \frac{q\{\alpha d(I_x,S_x)d(T_x,T_y) + \beta d(T_y,S_x)d(I_y,T_y) + \gamma d(I_x,T_y)\}^2}{\alpha d(T_x,T_y) + \beta d(I_y,S_x) + \gamma d(I_x,T_y)} \quad [1.2]$$

$$\forall\ \ \alpha \mathrm{d}(\mathrm{T_x},T_y) + \beta d(I_y,S_x) + \gamma d(I_x,I_y) \ne 0$$

$$0 \le q < 1,\ \ \alpha,\beta,\gamma \ge 0$$

Pandey & Dubey [6] obtained a unique common fixed point of three maps E,F&T satisfying.

$$d(E_x,F_y)\le\frac{\alpha_1 d(T_x,F_y)d(T_x,T_y)}{d(T_x,T_y)+d(T_x,F_y)}+\alpha_2\left[d(T_x,E_x)+d(T_y,F_y)\right]+\alpha_3\left[d(T_x,F_y)+d(T_y,F_x)\right]+\alpha_4 d(T_x,T_y) \quad [1.3]$$

Here we have generalized the contraction on five self maps as follows:

T_1, T_2, T_3, T_4 & T_5 on X to itself.

$$d(T_1,T_2x,T_3,T_4y)\le C_1\left[\frac{d(T_5x,T_1T_2x)d(T_5y,T_3T_4y)}{d(T_5x,T_5y)}\right]+C_2\left[\frac{d(T_5x,T_3T_4y)d(T_5y,T_3T_4y)}{d(T_1T_2x,T_3T_4y)}\right] \quad [1.4]$$

$$+C_3\left[d(T_5x,T_1T_2x)+d(T_5y,T_3T_4y)\right]+C_4\left[d(T_5x,T_3T_4y)+d(T_5y,T_1T_2x)\right]$$

$$+C_5\left[d(T_5x,T_5y)+d(T_1T_2x,T_3T_4y)\right]+C_6 d(T_5x,T_5y).$$

We see that under certain condition we get a unique common fixed point.

MAIN RESULTS

Let (X,d) be a complete metric space & $T_i : X \to X;\ i=1,2,3,4,5$ be five mapping satisfying [1.4] and [2.1] as follows

$$T_2x \ne T_5y;\quad \alpha_i \ge 0, \quad [2.1]$$

$$T_5(T_1T_2)=(T_1T_2)(T_5),\ T_5(T_3T_4)=(T_3T_4)(T_5)$$

$$T_1T_2=T_2T_1,\ T_3T_4=T_4T_3$$

$$T_1T_2(X)\subseteq T_5(X),\ T_3T_4(X)\subseteq T_5(X)$$

Then $T_i(i=1\ to\ 5)$ have a unique common fixed point in X.

Proof: Let $x_0, x_1, x_2 \in X$ and $T_1T_2(X)\subseteq T_5(X)$ such that

$$T_1T_2(x_0)=T_5(x_1),\ T_3T_4(x_1)=T_5(x_2)$$

$$T_3T_4(X)\subseteq T_5(X)$$

Since $T_1T_2x_{2n}=T_5x_{2n+1}$, $T_3T_4x_{2n+1}=T_5x_{2n+1}$, replace $x\to x_{2n}$ & $y\to x_{2n-1}$, then we have from [1.4]

$$d(T_5x_{2n+1},T_5x_{2n})=d(T_1T_2x_{2n},T_3T_4x_{2n-1}) \quad [2.3]$$

$$d(T_1T_2x_{2n},T_3T_4x_{2n-1})\le C_1\frac{d(T_5x_{2n},T_1T_2x_{2n})d(T_5x_{2n-1},T_3T_4x_{2n-1})}{d(T_5x_{2n},T_5x_{2n-1})}$$

$$+C_2\frac{d(T_5x_{2n},T_3T_4x_{2n-1})d(T_5x_{2n-1},T_3T_4x_{2n-1})}{d(T_1T_2x_{2n},T_3T_4x_{2n-1})}$$

$$+C_3\left[d(T_5x_{2n},T_1T_2x_{2n})+d(T_5x_{2n-1},T_3T_4x_{2n-1})\right]$$

$$+C_4\left[d(T_5x_{2n},T_3T_4x_{2n-1})+d(T_5x_{2n-1},T_1T_2x_{2n})\right]$$

$$+C_5\left[d(T_5x_{2n},T_5x_{2n-1})+d(T_1T_2x_{2n},T_3T_4x_{2n-1})\right]$$

$$+C_6d(T_5x_{2n},T_5x_{2n-1}).$$

Which yields from (2.2) and using property of metric space

$$d(T_5x_{2n+1},T_5x_{2n})\{1-C_1-C_3-C_5\}\leq d(T_5x_{2n-1},T_5x_{2n})(C_3+C_4+C_5)$$

Or

$$d(T_5x_{2n+1},T_5x_{2n})\leq Kd(T_5x_{2n},T_5x_{2n-1}) \qquad [2.4]$$

Where

$$0<K=\frac{(C_3+C_4+C_5)}{1-C_1-C_3-C_5}<1$$

Thus $[T_5x_{2n}]$ is a Cauchy Sequence in complete metric space X & have a common fixed point **u** in X such that

$$T_5x_{2n+1}=u=_{n\to\infty}T_5x_{2n} \qquad [2.5]$$

Now if $T_1T_2u\neq T_5u$ then

$$(T_1T_2u,T_5u)\leq\lim d(T_1T_2u,T_3T_4T_5x_{2n+1}) \qquad [2.6]$$

$$\leq C_1\ \frac{\lim d(T_5u,T_1T_2u)d(T_5T_5x_{2n+1},T_3T_4T_5x_{2n+1})}{d(T_5u_1,T_5T_5x_{2n+1})}$$

$$+C_2\frac{d(T_5u,T_3T_4T_5x_{2n+1})d(T_5T_5x_{2n+1},T_3T_4T_5x_{2n+1})}{d(T_1T_2u,T_3T_4T_5x_{2n+1})}$$

$$+C_3\left[d(T_5u,T_1T_2u)+d(T_5T_5x_{2n+1},T_3T_4T_5x_{2n+1})\right]$$

$$+C_4\left[d(T_5u,T_3T_4T_5x_{2n+1})+d(T_5T_5x_{2n+1},T_1T_2u)\right]$$

$$+C_5\left[d(T_5u,T_5T_5x_{2n+1})d(T_1T_2u,T_3T_4T_5x_{2n+1})\right]+C_6\left[d(T_5u,T_5T_5x_{2n+1})\right]$$

which reduced to

$d(T_1T_2u, T_5u)(1 - C_5 - C_4 - C_3) \le 0$.

$$d(T_1T_2u, T_5u) \le 0 \qquad [2.7]$$

which is a Contraction therefore

$T_1T_2(u) = T_5(u),$ also we have $T_5(u) = T_3T_4(u),$ hence

$$T_1T_2(u) = T_5(u) = T_3T_4(u), \text{ and} \qquad [2.8]$$

$$T_5(T_5u) = T_5(T_1T_2u) = T_1T_2(T_1T_2u) = T_1T_2(T_3T_4u) = T_1T_2(T_5u) = T_5(T_3T_4u)$$
$$= T_3T_4(T_5u) = T_3T_4(T_1T_2u) = T_3T_4(T_3T_4u) \qquad [2.9]$$

Now if $T_1T_2u \ne T_3T_4(T_1T_2u)$

$$d(T_1T_2u, T_3T_4(T_1T_2u)) \le C_1 \frac{d(T_5u, T_1T_2u)d(T_5T_1T_2u, T_3T_4T_3T_4u)}{d(T_5u, T_5T_3T_4u)} \qquad [2.10]$$

$$+ C_2 \frac{d(T_5u, T_3T_4T_3T_4u)d(T_5T_3T_4u, T_3T_4T_3T_4u)}{d(T_1T_2u, T_3T_4T_3T_4u)}$$

$$+ C_3\left[d(T_5u, T_1T_2u) + d(T_5T_3T_4u, T_3T_4T_3T_4u)\right]$$

$$+ C_4\left[d(T_5u, T_3T_4T_3T_4u) + d(T_5T_3T_4u, T_1T_2u)\right]$$

$$+ C_5\left[d(T_5u, T_5T_3T_4u) + d(, T_1T_2u, T_3T_4T_3T_4u)\right] + C_6 d(T_5u, T_5T_3T_4u).$$

Therefore (2.10) reduces to

$$d(T_5u, T_5T_5u)(1 - 2C_4 - 2C_5 - C_6) \le 0$$

Or equivalently

$$d(T_5u, T_5T_5u) \le 0 \qquad [2.11]$$

which is a Contraction, therefore

$$T_5(T_5(u)) = T_5(u)$$

Also

$$T_3T_4(T_1T_2u) = T_1T_2u$$

Thus we get

$$T_1T_2u = T_3T_4(T_1T_2u) = T_5(T_1T_2u)$$

$$= T_1T_2(T_1T_2(T_1T_2u)).$$

Thus $T_1T_2u = v$ is a common fixed point of $T_5T_1T_2 \& T_3T_4$, if possible we take another common fixed point of $T_1T_2T_5 \& T_3T_4$.Thus

$$[d(v,w)] = [d(T_1T_2v, T_3T_4w)]$$

which is a view of [1.4] yields [2.12]

$$d(v,w) \le C_1 \frac{[d(T_5v, T_1T_2v)]d(T_5w, T_5T_4w)}{d(T_5v, T_5w)} + C_2 \frac{[d(T_5v, T_3T_4w)d(T_5w, T_3T_4w)]}{d(T_1T_2v, T_3T_4w)}$$
$$+ C_3[d(T_5v, T_3T_4v) + d(T_5w, T_3T_2w)] + C_4[d(T_5v, T_3T_4w) + d(T_5w, T_1T_2v)]$$
$$+ C_5[d(T_5v, T_5w) + d(T_1T_2v, T_3T_4w)] + C_6 d(T_5v, T_5w)$$

Equation [2.12] reduced to

$$d(v,w)(1 - 2C_4 - 2C_5 - C_6) \le 0$$

Or equivalently

$$d(v,w) = 0 \Rightarrow v = w$$

Now to show that v is a unique common fixed point of $T_1, T_2, T_3, T_4 \& T_5$ we find that

$$T_1(v) = T_1(T_1T_2v) = T_1(T_2T_1v) = T_1T_2(T_1v) \Rightarrow T_1v$$

which implies that T_1v is as another fixed point of $T_1 \& T_2$.Hence by uniqueness of fixed point of $T_1 \& T_2$, we obtain $\mathbf{v}$ a fixed point of T_1, similarly a unique fixed point of T_2, T_3, T_4 $\& T_5$.Hence proved.

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